Optimal Resource Allocation in Packet Networks that use Rate Based Schedulers

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ABSTRACT

Provision of Quality-of-Service guarantees is an important issue in the design of Integrated Services packet networks. A key challenge is to support a large number of sessions with different performance requirements, while minimizing the cost as measured by the network resources. In this paper, we investigate the problem of optimal nodal bandwidth allocation for a connection to satisfy its end-to-end delay requirement in a network of rate-based schedulers. We show that a less-conservative resource allocation can be obtained by taking advantage of the properties of the schedulers and the inter-hop dependencies of traffic streams than by considering upper bounds to these traffic streams. We also characterize the optimal bandwidth allocation at each node that satisfies an end-to-end delay requirement in a network of Rate Proportional Servers.

I. INTRODUCTION

Broadband integrated services networks are expected to support a variety of traffic types with diverse Quality-of-Service requirements. A key issue is how to provide the resources in order to meet the requirements of each connection. Establishing QoS connections in a way that minimizes the network resources used is an important network optimization problem. One of the problems, in this context, is to map the end-to-end QoS requirements into local requirements. Addressing this problem impacts both the routing process and allocation of the resources on the selected route.

In this paper, we investigate the problem of optimal nodal bandwidth allocation to satisfy an end-to-end delay requirement in a network of rate-based schedulers. A similar problem was investigated in [10]. Their focus was on loss rate guarantees. A major contribution of [10] was the development of a nodal metric that predicts the relative performance of QoS allocation policies in a network setting. [9] investigated the problem of optimal resource allocation for end-to-end QoS requirements. It associated with each link a convex cost function and established a polynomial solution to partition an end-to-end QoS requirement into nodal requirements such that the overall cost function is minimized. However, both the above studies assume unmodified connection characteristics in the network and do not incorporate the inter-hop dependencies involved along a session's route. This could result in an overly conservative resource allocation. Another set of related studies is [11] and [7] which investigated QoS partitioning and routing for connections in networks with uncertain parameters.

II. SUMMARY OF CONTRIBUTIONS

In the present work we show:

- The characterization of the departure process of fluid Rate Proportional Servers.
- A better resource allocation (less conservative) can be obtained by taking advantage of the properties of the schedulers and the inter-hop dependencies of traffic streams than by considering upper bounds to these traffic streams.
- The optimal bandwidth allocation at each node that satisfies an end-to-end delay requirement in a network of *Rate Proportional Servers*.

III. PRELIMINARIES

Leaky Bucket characterization of a traffic stream is based on specifying a two parameter (σ, ρ) envelope on the volume of the arriving traffic [2].

Definition 1: Given $\sigma > 0$, and $\rho > 0$, the traffic stream $A(t)$ is called $(\sigma, \rho) - regular$, if for any interval $[t_1, t_2)$

$$A[t_1, t_2) \leq \sigma + \rho(t_2 - t_1)$$

Let g_i be the rate allotted to session i.

Definition 2: A session i busy period is a maximum interval of time $(\tau_1, \tau_2]$ such that at an time $t \in (\tau_1, \tau_2]$, the accumulated arrivals of session i since the beginning of the interval do not fall below the total service received during the interval at a rate exactly g_i. That is:

$$A_i(\tau_1, t) \geq g_i(t - \tau_1)$$

In this paper we assume that:

- Arrival processes are leaky bucket constrained.
- The scheduling discipline at each node belongs to the family of disciplines referred to as *Rate Proportional Schedulers* (RPS)[14]. GPS and Fluid Virtual Clock are examples of Fluid RPS, while their packetized versions: PGPS [12] and Virtual Clock [16], are examples of Packet-by-Packet RPS (PRPS).

IV. OPTIMAL RESOURCE ALLOCATION WITH WORST CASE ARRIVAL PROCESSES INTO EACH NODE

The arrival process of session k into the network is $(\sigma_k, \rho_k) - regular$. The departure process was shown to be $(\sigma_k, \rho_k) - regular$ for the GPS discipline in [13]. The following theorem generalizes this and shows that the output process remains $(\sigma_k, \rho_k) - regular$ for the whole family of RPS scheduling disciplines. The proofs of all the Theorems and

Lemmas are in the Appendix.

Theorem 1: If the arrival process into a node with a fluid RPS is $(\sigma_k, \rho_k) - regular$, then so is the departure process.

Consider a connection k that is $(\sigma_k, \rho_k) - regular$ upon entry into the network, and requires an end-to-end delay guarantee of D. A natural question in this context is: how should the demanded end-to-end delay guarantee D be split among the nodes in the connection's path so that the network resources (i.e. bandwidth) allocated is the least? To address this question we consider a bandwidth-delay curve at node i denoted as $R_i(d_i)$: $R_i(d_i)$ gives the bandwidth that must be allocated to connection k at node i so that the delay experienced at that node is never greater than d_i. Note that the bandwidth to be allocated at node i would depend upon the arrival process into node i. In general, the characterization of the arrival process into node i could change with i, and this explains the notation $R_i(.)$. $R_i(.)$ is assumed to be a convex decreasing function. This is motivated by the reciprocal relationship between bandwidth and delay. Thus, the problem is:

$$Minimize \quad \sum_{i=1}^{n} R_i(d_i)$$

$$subject\ to \quad \sum_{i=1}^{n} d_i \leq D$$

Theorem 1 asserts that the arrival process into node i can be characterized as $(\sigma_k, \rho_k) - regular$ irrespective of i. Hence

$$R_i(d_i) = R(d_i), \quad i = 1, \ldots, n.$$

Exploiting the convex decreasing nature of the resource-delay curve $R(.)$, it can be shown very simply that the total resource allocation is minimized when D is split equally among the nodes. This is stated below.

Theorem 2: If a $(\sigma_k, \rho_k) - regular$ stream arrives to a tandem of n RPS nodes with an end-to-end delay requirement of D, then the amount of allocated network resources is minimized when each RPS node allocates a rate sufficient to guarantee a nodal delay of D/n.

Corollary 2.1: It is optimal to split the worst case end-to-end delay requirement equally among the nodes in the path.

Consider a connection k with $(\sigma_k, \rho_k) = (40, 10)$ and an end-to-end delay requirement $D = 2$, passing through a network of two nodes. Using Theorem 2, the optimal split is 1 at each of the nodes and the total bandwidth required is equal to 80. The next Section shows how this allocation can be improved.

V. OPTIMAL RESOURCE ALLOCATION WITH COUPLED ARRIVAL PROCESSES

Theorem 1 provides a *worst case* characterization of the arrival processes into node $i,\ i = 1, \ldots, n$. According to this result, the arrival process into any node in the tandem can be as bursty as the arrival process at the ingress of the network. However, the simple act of passing through a RPS can smoothen out a bursty process. This Section considers this potential smoothing and arrives at a less conservative allocation. From now on, we consider that the arrival process into the network is greedy [12] from time zero, i.e.

$$A_k(0, t) = \sigma_k + \rho_k t, \quad \forall t \geq 0$$

A. Fluid RPSs

Let g_i be the guaranteed rate allotted to connection k at node i. This means that while all connections passing through node i are backlogged, connection k is served at rate g_i. Now, consider the guaranteed rate g_{i+1} allotted to connection k at the next node. If $g_{i+1} > g_i$, the resource allocation is wasteful because there may not be enough arrivals to use the allocated rate g_{i+1}. Hence $g_{i+1} \leq g_i$ is a reasonable allocation policy. Repeating the argument for all nodes in the tandem we arrive at the following constraint:

$$g_1 \geq g_2 \geq \cdots \geq g_n > \rho_k \tag{1}$$

Then the following can be shown:

Proposition 1: If,
(i) connection k is $(\sigma_k, \rho_k) - regular$ upon entry into a tandem of n RPS nodes,
(ii) g_i is the guaranteed rate allocated to connection k at the i^{th} node, $i = 1, \ldots, n$,
(iii) $g_1 \geq g_2 \geq \cdots \geq g_n > \rho_k$,
then the maximum worst case delay is experienced by the last bit of the initial burst.

Considering the sample path of a greedy process and Eqn.(1), it can be shown that:
$D = \sigma_k / g_n$, where D = end-to-end delay requirement of connection k.
Then the problem of minimizing allocated resources is:
Minimize $(g_1 + g_2 + \cdots + g_n)$
subject to (a) $\sigma_k / g_n = D$ and
(b) $g_1 \geq g_2 \geq \cdots \geq g_n > \rho_k$
Clearly, the optimal resource allocation is $g_i = \sigma_k / D, \ i = 1, \cdots, n$.

Theorem 3: For a greedy $(\sigma_k, \rho_k) - regular$ source with an end-to-end delay requirement D, arriving to a tandem of n fluid RPSs, the optimal resource allocation is $g_i = \sigma_k / D,\ i = 1, \ldots, n$.

Theorem 3 can be thought as a refinement of Theorem 2. The results of Theorem 3 are obtained by considering explicitly the actual arrival process into the nodes, while those of Theorem 2 are based on looser upper bounds. As a result, Theorem 2 drastically over-estimates the resources to be allocated. Secondly, it is interesting to note the following as a consequence of Theorem 3.

Corollary 3.1: The "improved" optimal resource allocation results of Theorem 3 imply that the optimal delay split is not the equal split; rather, the optimal delay split results when the entire end-to-end delay is allotted to the first node.

For the example considered in Section IV i.e. a connection k with $(\sigma_k, \rho_k) = (40, 10)$ and an end-to-end delay requirement

$D = 2$, passing through a network of two nodes, Theorem 3 gives the total bandwidth required to be equal to $2\,\sigma_k/D = 40$.

So far, we have considered fluid arrivals and a fluid server. In practice, of course, we have packet arrivals and a packet server. We approach the realistic situation of packet arrivals and packet server by considering the intermediate scenario of packet arrivals and a fluid server.

The case of packet arrivals to a fluid server is characterized by the fact that the service of a packet does not begin till the last bit of the packet has arrived. This implies an additional *packetization* delay at each node. If L_k is the maximum packet length of session k and g_i is the guaranteed rate given to session k at node i, then the packetization delay experienced at node $(i+1)$ is L_k/g_i. The additional packetization delays appear in the formulation as follows:

Minimize $(g_1 + g_2 + \cdots + g_n)$
subject to $(a)\ \frac{\sigma_k}{g_n} + \frac{L_k}{g_1} + \cdots + \frac{L_k}{g_{n-1}} \leq D$
and $(b)\ g_1 \geq g_2 \geq \cdots \geq g_n$

Theorem 4: The optimal solution to the above problem is:

$$g_i = \frac{\sigma_k}{D}\left[1 + \frac{(n-1)L_k}{\sigma_k}\right] \quad for\ 1 \leq i \leq n$$

Remark: We observe that on putting $L_k = 0$ in the result of Theorem 4, the result of Theorem 3 is recovered as expected.

B. Packet Servers

In this section, we do not consider general packet-by-packet RPS's. The specific scheduling disciplines considered are PGPS, $(WF)^2Q$ [1], and Virtual Clock [16]. In packet servers, only one session can be served at each time-instant and preemption is not possible. It is shown in [12] that for PGPS server, the additional delay incurred by a packet in a packet server (compared to that in a fluid server) is at most L_{max}/C_i where L_{max} is the maximum packet size among all the connections and C_i is the capacity of node i. Similar results have been proved in [1] for $(WF)^2Q$ and in [6] for Virtual Clock.†

The optimization problem in this case is:
Minimize $(g_1 + g_2 + \cdots + g_n)$
subject to $(a)\ \frac{\sigma_k}{g_n} + \frac{L_k}{g_1} + \cdots + \frac{L_k}{g_{n-1}} + \frac{L_{max}}{C_1} + \cdots + \frac{L_{max}}{C_n} \leq D$
and $(b)\ g_1 \geq g_2 \geq \cdots \geq g_n$

Theorem 5: The optimal resource allocation for a session k in a network of PGPS, $(WF)^2Q$, and Virtual Clock servers to satisfy an end-to-end delay bound of D is:

$$g_i = \frac{\sigma_k + (n-1)L_k}{D - \sum_{i=1}^{n} \frac{L_{max}}{C_i}} \quad for\ \ 1 \leq i \leq n$$

VI. CONCLUSION

In this work we addressed the problem of optimizing the total allocated resources in a network of RPS servers while establishing a connection with an end-to-end delay requirement.

In this work we assumed that a session's service rate at a node remains constant as long as it is backlogged. However, in reality a session's service rate could increase depending on the backlog clearing times of the other sessions at the same node. Our current research focuses on obtaining a better resource allocation by taking into consideration such interactions among the sessions at a node.

Appendix

Proof of Theorem 1: Let τ be the beginning of a session k busy period. Let $W_k(\tau, t)$ be the total number of bits transmitted during the interval $(\tau, t]$, from session k.

Then, at any time $t > \tau$ that belongs to the same busy period, the service offered to session k is (Theorem 2 in [14]),

$$W_k(\tau, t) \geq g_k(t - \tau) \tag{2}$$

Let $Q_k(t)$ denote the backlog of session k at time t. Then:

$$Q_k(t) = A_k(\tau, t) - W_k(\tau, t) \tag{3}$$

Further, let $Q_k(t) \leq Q_k^{max}$ for any (σ_k, ρ_k)-regular stream $A_k(t)$. Then, it was shown in Lemma 2 of [4] that,

$$W_k(\tau, t)\ \ is\ \ (Q_k^{max}, \rho_k) - regular \tag{4}$$

Equation (4) makes no assumptions about the arriving traffic of the other sessions or about the service discipline at the multiplexer. Suppose Q_k^{max} is achieved at some time t_1, then for a RPS, we get from (2) and (3)

$$\begin{aligned} Q_k^{max} = Q_k(t_1) &\leq \sigma_k + (\rho_k - g_k)(t_1 - \tau) \\ &\leq \sigma_k \end{aligned}$$

Hence from (4), the departure process $W_k(\tau, t)\ \ is\ \ (\sigma_k, \rho_k) - regular$

Proof of Theorem 2: As mentioned before, the bandwidth-delay curve $R(.)$ is assumed to be a convex function. Thus we have:

$$R(\frac{d_1}{n} + \ldots + \frac{d_n}{n}) \leq \frac{R(d_1)}{n} + \cdots + \frac{R(d_n)}{n}$$

$$n * R(\frac{D}{n}) \leq R(d_1) + \cdots + R(d_n)$$

Thus the minimized total allocated resources $= n * R(\frac{D}{n})$.

Proof of Proposition 1: Let $W_k^i(t)$ denote the total number of bits from session k transmitted in $[0, t]$ at node i. Then, as illustrated in Fig.(1) the output traffic characterization of session k at node 1 is:

$$W_k^1(t) = \min(\sigma_k + \rho_k t, g_1 t)$$

At node 2 it is:

$$\begin{aligned} W_k^2(t) &= \min(W_k^1(t), g_2 t) \\ &= \min(\sigma_k + \rho_k t, g_2 t)\ \ (From Eqn.1\ and\ 2) \end{aligned}$$

And thus at node i we have:

$$\begin{aligned} W_k^i(t) &= \min(W_k^{i-1}(t), g_k t) \\ &= \min(\sigma_k + \rho_k t, g_k t) \end{aligned} \tag{5}$$

Now, consider the sample path of the last bit of the burst of session k, under the traffic characterization in (5). It can be seen from Fig.(1) that:

†A bound on the worst case delay introduced by a packet server for a *general packetized* RPS does not exist [14]. Hence the results of this section are specific to certain scheduling disciplines and do not apply to a general PRPS.

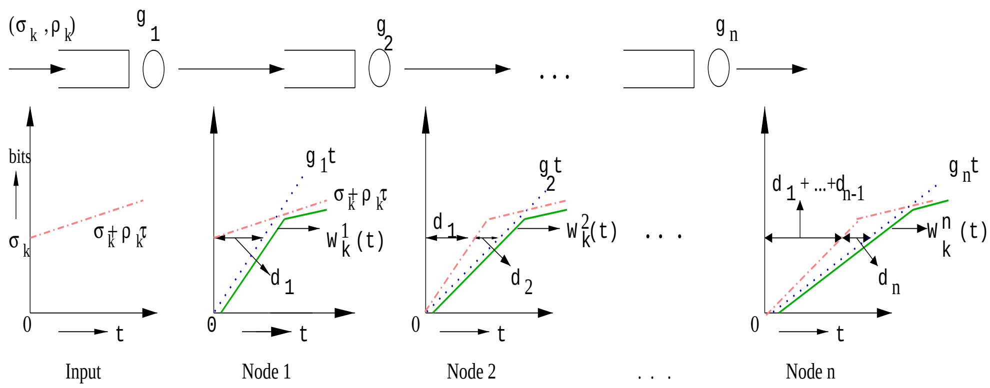

Fig. 1. Traffic Characterization in the network

$$d_1 = \frac{\sigma_k}{g_1} \quad or \quad g_1 = \frac{\sigma_k}{d_1} \tag{6}$$

$$d_2 = \frac{(g_1 - g_2)d_1}{g_2} \quad or \quad g_2 = \frac{\sigma_k}{d_1 + d_2} \tag{7}$$

$$\vdots$$

$$d_n = \frac{(g_{n-1} - g_n)(d_1 + \cdot\cdot + d_{n-1})}{g_n} \quad or$$

$$g_n = \frac{\sigma_k}{d_1 + \cdots + d_n} \tag{8}$$

From (8) the end-to-end delay experienced by the last bit of the initial burst is:

$$d_1 + d_2 + \cdots + d_n = \frac{\sigma_k}{g_n} \tag{9}$$

Now, we have to show that the delay of any other bit is $< \frac{\sigma_k}{g_n}$. Consider the sample path of a bit within the initial burst. Let the backlog seen by this bit at the first node be $\sigma_1 (< \sigma_k)$. Then, the equations (6), (7), (8) and (9) also hold true for this bit, with the σ_k replaced by σ_1. The end-to-end delay experienced by this bit is $\frac{\sigma_1}{g_n}$ which is $< \frac{\sigma_k}{g_n}$.

In the final case, consider the sample path of a bit that arrives at the first node at some epoch $t_1 > 0$, i.e. at an epoch after the last bit of the initial burst arrives. It can be easily seen that if such a bit experiences a positive delay at some node i, then it must experience positive delays at the subsequent nodes $i,\ i+1, \ldots, n$ too, where $i \geq 1$. This is because of (1). Then if i is the first node where such a bit experiences a non-zero delay we have:

$$\begin{aligned} \sigma_k + \rho_k t_1 &= (t_1 + d_i)g_i \;; i \ \geq 1 \\ &= (t_1 + d_i + d_{i+1})g_{i+1} \\ &\ \vdots \\ &= (t_1 + d_i + \cdots + d_n)g_n \end{aligned}$$

The end-to-end delay of this bit is:

$$\begin{aligned} d_i + d_{i+1} + \cdots + d_n &= \frac{\sigma_k - (g_n - \rho_k)\ t_1}{g_n} \\ &< \frac{\sigma_k}{g_n} \end{aligned}$$

Thus under the conditions $(i), (ii)\ and\ (iii)$ of Proposition 1, the maximum worst-case delay is experienced by the last bit of the initial burst.

Proof of Theorem 3: It was shown in the proof of *Proposition 1* that the end-to-end delay, $D = \frac{\sigma_k}{g_n}$. This equation along with the condition $g_1 \geq g_2 \geq \cdots \geq g_n > \rho_k$ gives the optimal resource allocation as:

$$g_i = \frac{\sigma_k}{D}, \ \ i = 1, \ldots, n$$

Proof of Theorem 4: Let $(1 - \frac{L_k}{\sigma_k}) = a$. The Optimization problem can also be written as follows:

Minimize: $-(\frac{\sigma_k}{d_1} + \frac{\sigma_k}{d_1 a + d_2} \cdots + \frac{\sigma_k}{d_1 a^{n-1} + d_2 a^{n-2} + \cdots + d_{n-1} a + d_n})$
subject to:

$$D - d_1 - d_2 - \ldots - d_n \geq 0 \tag{10}$$

$$d_1 \geq 0 \tag{11}$$

$$d_2 + (a-1)d_1 \geq 0 \tag{12}$$

$$\vdots$$

$$d_1 a^{n-2}(a-1) + d_2 a^{n-3}(a-1) + \cdots + d_{n-1}(a-1) + d_n \geq 0 \tag{13}$$

where the set of constraints in (12), (13) are due to the constraints $g_1 \geq g_2 \geq \cdots \geq g_n$.

In order to obtain the optimal solution we use the Theorem of *Kuhn Tucker* which is stated below:

Theorem of Kuhn and Tucker: Let f be a concave function mapping U into $\mathcal{R}$ where $U \subset \mathcal{R}^n$ is open and convex. For $i = 1, 2, \ldots, l$, let $h_i : U- > \mathcal{R}$ also be concave functions. Suppose there is some $\bar{x} \in U$ such that:

$$h_i(\bar{x}) > 0, \ \ i = 1, \ldots, l$$

Then x^* maximizes f over:

$$\mathcal{D} = \{x \in U | h_i(x) \geq 0, \ \ i = 1, \ldots, l\}$$

if and only if there is $\lambda^* \in \mathcal{R}^l$ such that the Kuhn-Tucker first-order conditions hold:

$$[KT - I] \ \ Df(x^*) + \sum_{i=1}^{l} \lambda_i^* Dh_i(x^*) = 0$$

$$[KT - II] \ \ \lambda^* \geq 0, \ \ \sum_{i=1}^{l} \lambda_i^* h_i(x^*) = 0$$

The objective function and the constraints in (10) . . . (13) satisfy the conditions in the *Kuhn Tucker Theorem*. We find

that if:
$(d_1^*, d_2^*, \ldots, d_n^*) = (\frac{\sigma_k D}{\sigma_k+(n-1)L_k}, \frac{L_k D}{\sigma_k+(n-1)L_k}, \cdots \frac{L_k D}{\sigma_k+(n-1)L_k})$

then $\exists(\lambda_1^*, \ldots, \lambda_{n+1}^*) \in \mathcal{R}^{n+1}$ such that the *Kuhn Tucker (KT)* first-order conditions hold. The $(\lambda_1^*, \ldots, \lambda_{n+1}^*)$ which satisfy the first-order KT equations are:

$$\begin{aligned}\lambda_1^* &= n\sigma_k(\frac{1+(n-1)\frac{L_k}{\sigma_k}}{D^2}) \\ \lambda_2^* &= 0 \\ \lambda_j^* &= (j-2)(\frac{1+(n-1)\frac{L_k}{\sigma_k}}{D^2})(\sigma_k - L_i);\ \ 3 \le j \le (n+1)\end{aligned}$$

Thus from the *KT* Theorem we know that $(d_1^*, d_2^*, \ldots, d_n^*) = (\frac{\sigma_k D}{\sigma_k+(n-1)L_k}, \frac{L_k D}{\sigma_k+(n-1)L_k}, \ldots, \frac{L_k D}{\sigma_k+(n-1)L_k})$ maximizes the objective function.
On substitution of $(d_1^*, d_2^*, \ldots, d_n^*)$ in the objective function, we obtain the minimum resource allocation to be:

$$g_i = \frac{\sigma_k}{D}[1 + \frac{(n-1)L_k}{\sigma_k}]\ for\ 1 \le i \le n$$

Proof of Theorem 5: Let $(1 - \frac{L_k}{\sigma_k}) = a$. The optimization problem in this case may be written as:

Minimize: $-(\frac{\sigma_k}{d_1 - \frac{L_{max}}{C_1}} + \frac{\sigma_k}{(d_1 - \frac{L_{max}}{C_1})a + (d_2 - \frac{L_{max}}{C_2})} + \cdots$
$+ \frac{\sigma_k}{(d_1 - \frac{L_{max}}{C_1})a^{n-1} + \cdots + (d_n - \frac{L_{max}}{C_n})}$

subject to:

$D - d_1 - d_2 - \ldots - d_n \ge 0$

$(d_1 - \frac{L_{max}}{C_1}) \ge 0$

$(d_2 - \frac{L_{max}}{C_2}) + (a-1)(d_1 - \frac{L_{max}}{C_1}) \ge 0$

$\vdots$

$(d_1 - \frac{L_{max}}{C_1})a^{n-2}(a-1) + \cdots + (d_{n-1} - \frac{L_{max}}{C_{n-1}})(a-1) + (d_n - \frac{L_{max}}{C_n}) \ge 0$

On substituting $d_i - \frac{L_{max}}{C_1} = d_i^{'};\ \ 1 \le i \le n$, the above problem is of the same form as in Theorem 4. Thus the proof of this problem is along the same lines as that of Theorem 4 and we obtain the optimal split as:

$$\begin{aligned}d_1 &= \frac{\sigma_k(D - \sum_{j=1}^n L_{max}C_j)}{\sigma_k + (n-1)L_k} + \frac{L_{max}}{C_1} \\ d_i &= \frac{L_k(D - \sum_{j=1}^n L_{max}C_j)}{\sigma_k + (n-1)L_k} + \frac{L_{max}}{C_i};\ \ 2 \le i \le n\end{aligned}$$

On substitution of $(d_1^*, d_2^*, \ldots, d_n^*)$ in the objective function, we obtain the minimum resource allocation to be:

$$g_i = \frac{\sigma_k + (n-1)L_k}{D - \sum_{j=1}^n \frac{L_{max}}{C_j}}; 1 \le i \le n$$

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