Definitions and Traffic Models

B.1 Definitions

In this thesis, we are interested in deterministic performance guarantees. However, in some instances we refer to terms that are concerned with statistical performance guarantees informally. We define these terms rigorously in this section.

We assume that time is slotted into cell times. Let $A_{i,j}(n)$ denote the cumulative number of arrivals to input $i$ of cells destined to output $j$ at time $n$. Let $A_i(n)$ denote the cumulative number of arrivals to input $i$. During each cell time, at most one cell can arrive at each input. $\lambda_{i,j}$ is the arrival rate of $A_{i,j}(n)$. $D_{i,j}(n)$ is the cumulative number of departures from output $j$ of cells that arrived from input $i$, while $D_j(n)$ is the aggregate number of departures from output $j$. Similarly, during each cell time, at most one cell can depart from each output. $X_{i,j}(n)$ is the total number of cells from input $i$ to output $j$ still in the system at time $n$. The evolution of cells from input $i$ to output $j$ can be represented as:

$$X_{i,j}(n+1) = X_{i,j}(n) + A_{i,j}(n) - D_{i,j}(n). \quad (B.1)$$

Let $A(n)$ denote the vector of all arrivals $\{A_{i,j}(n)\}$, $D(n)$ denote the vector of all departures $\{D_{i,j}(n)\}$, and $X(n)$ denote the vector of the number of cells still in the system.
system. With this notation, the evolution of the system can be described as

\[ X(n + 1) = X(n) + A(n) - D(n). \]  

**Definition B.1. Admissible:** An arrival process is said to be admissible when no input or output is oversubscribed, i.e., when \( \sum_i \lambda_{i,j} < 1 \), \( \sum_j \lambda_{i,j} < 1 \), \( \lambda_{i,j} \geq 0 \).

**Definition B.2. IID:** Traffic is called independent and identically distributed (iid) if and only if:

1. Every arrival is independent of all other arrivals both at the same input and at different inputs.
2. All arrivals at each input are identically distributed.
Definition B.3. 100% throughput: A router is said to achieve 100% throughput if under any admissible iid traffic, for every $\epsilon > 0$, there exists $B > 0$ such that

$$
\lim_{n \to \infty} Pr\{\sum_{i,j} X_{i,j}(n) > B\} < \epsilon.
$$