Characterization of Networks Supporting Multi-dimensional Linear Interval Routing Schemes

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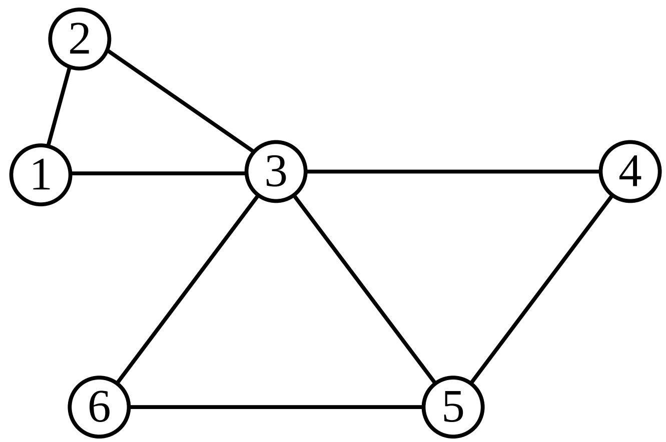
Overview

1. The routing problem and routing schemes
2. Interval routing schemes
3. Multi-dimensional interval routing schemes (MIRS)
4. Characterization of networks supporting MLIRS
    - Necessary conditions
    - Sufficient conditions
5. Conclusion

The Routing Problem

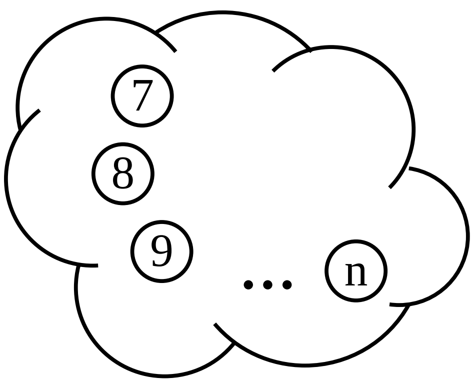


The Classical Solution: Routing Tables

Destination	Forward to
1	3
2	3
3	3
4	--
5	5
6	5

Interval Routing Scheme

Destination	Forward to
[1..3]	3
[5..n]	5

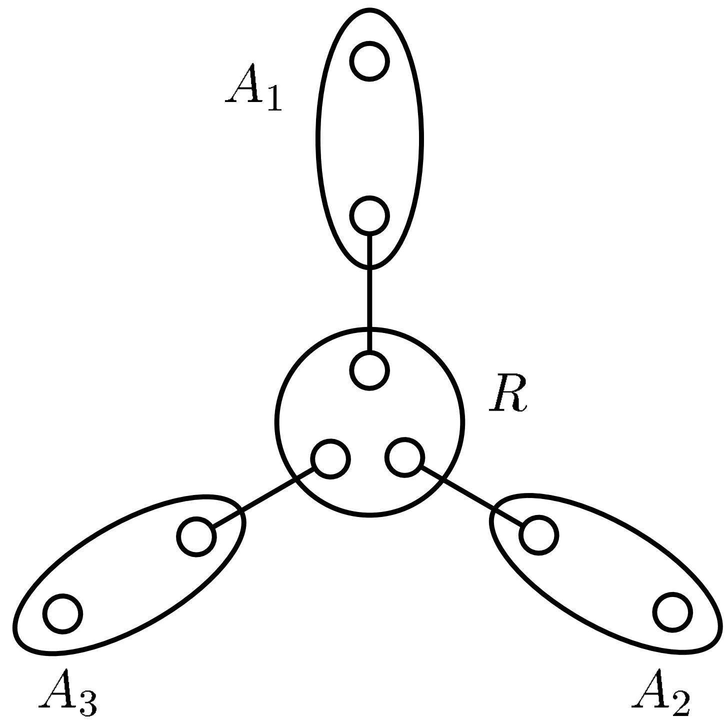


[Santoro and Khatib, 85]

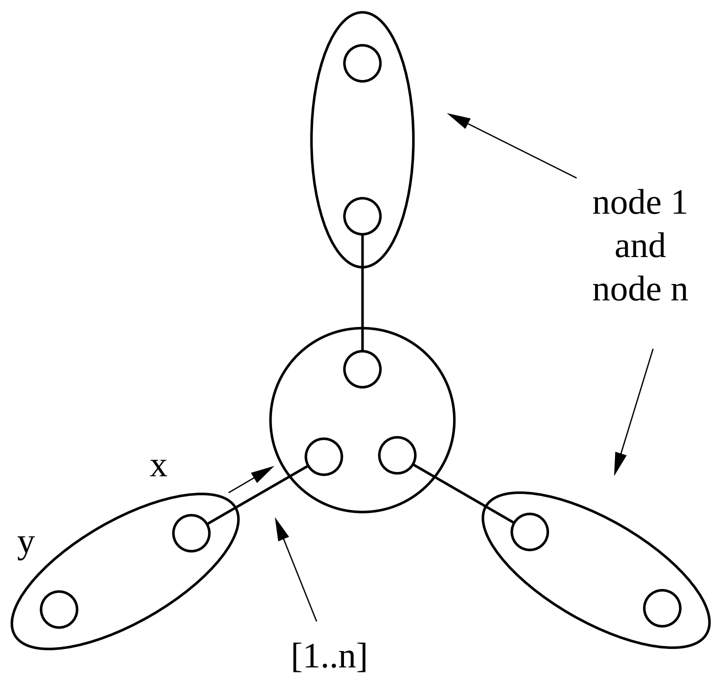
Characterization Problem

A 3-windmill graph:

- has 4 parts (3 *arms* and a *center* R);
- each arm has at least 2 nodes;
- there is no link connecting arms;
- each arm is connected to R by a unique link.



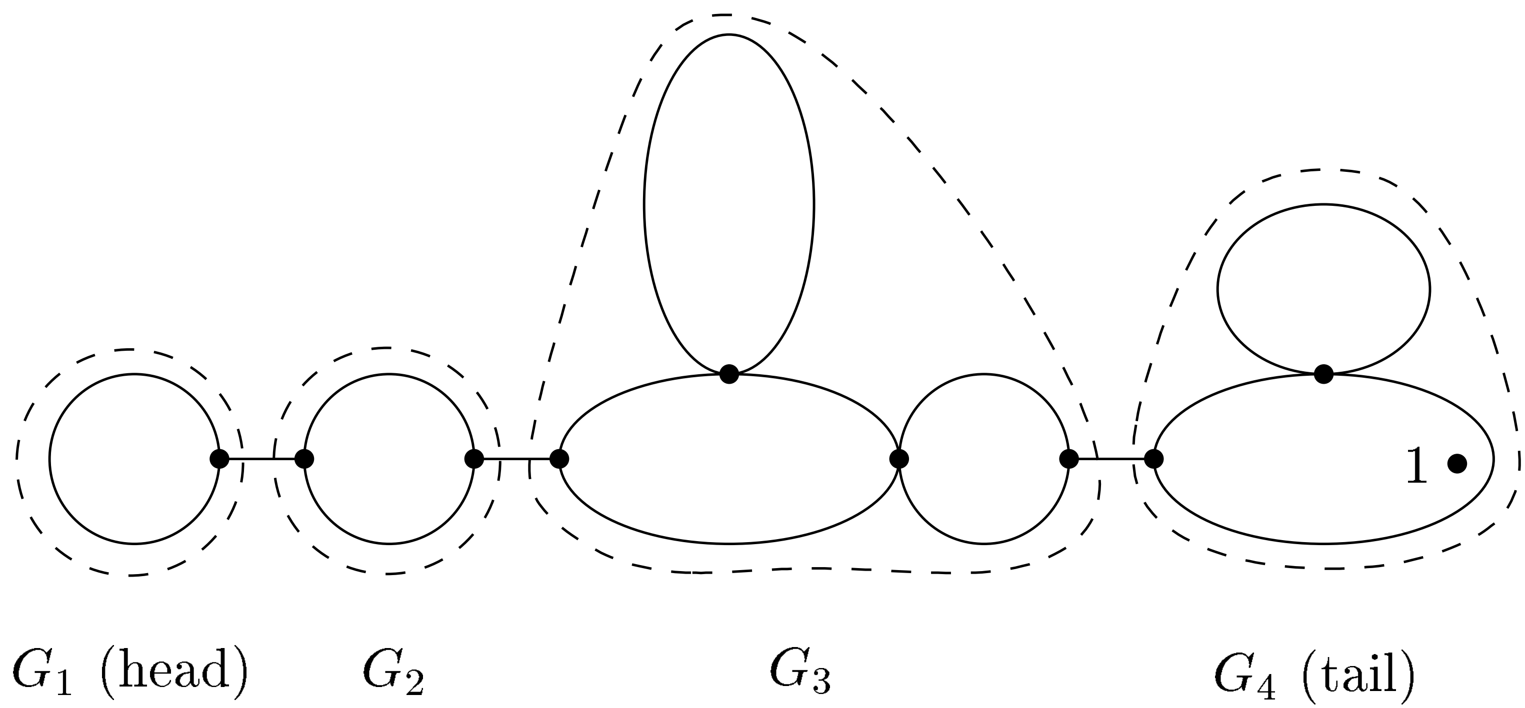
Characterization Problem: Cont'd



A 3-windmill network does not have an IRS.

Characterization Problem: Cont'd

Any graph which is not a 3-windmill graph (chain) has an IRS [Fraigniaud and Gavoille, 94].



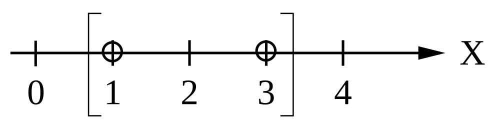
Characterization Problem: Cont'd

How to expand the class of networks which support an IRS?

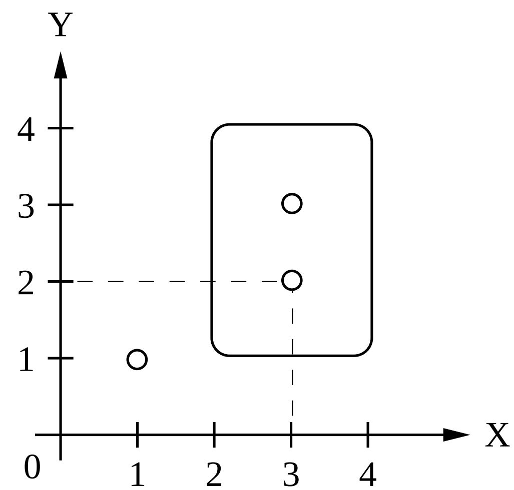
- We can assign *two or more intervals* to each link [van Leeuwen and Tan, 87].
- We can use *multi-dimensional labels* [Flammini *et al.*, 98]. ⋆⋆⋆

Multi-dimensional Interval Routing Schemes

- Node labels: A list of d integers, *e.g.* (3,2)
- Link labels: A list of d intervals, *e.g.* [2..4, 1..2]



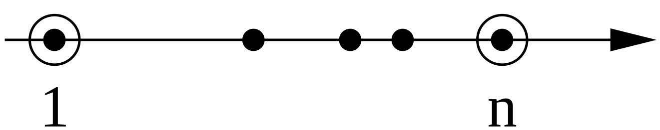
1-dimensional IRS

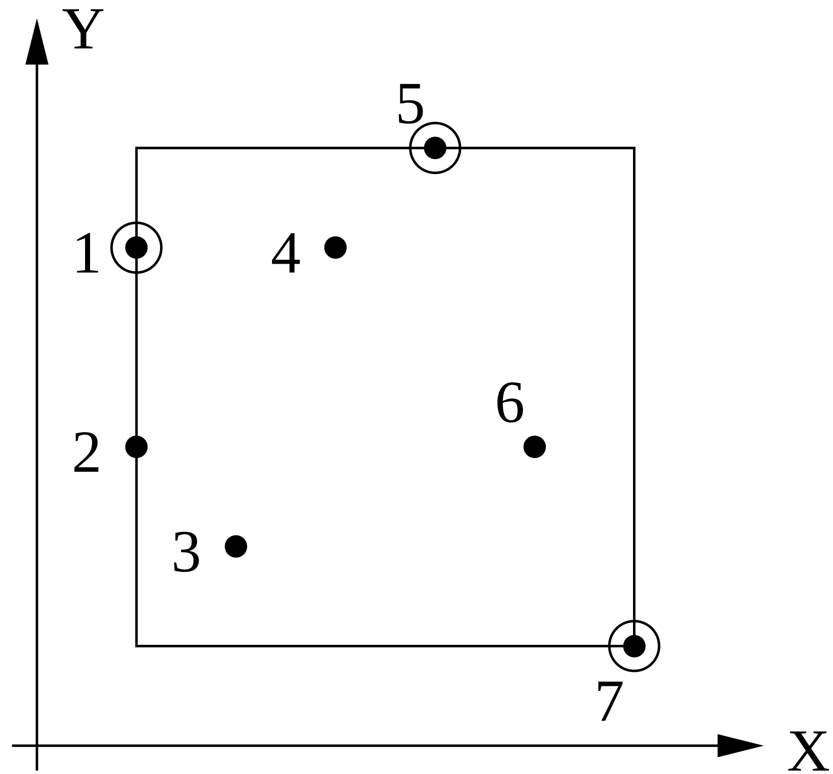


2-dimensional IRS

A d-dimensional MIRS requires $\theta(d\Delta n \log n)$ space.

The Boundary Set

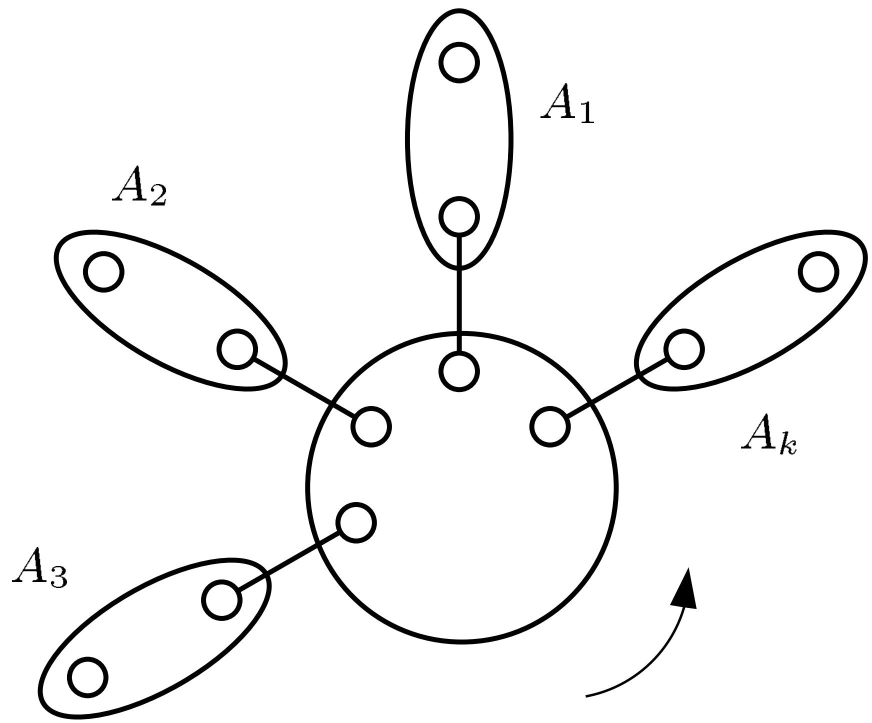




Any interval containing the boundary set contains all other points.

Characterization Problem: Cont'd

A k-windmill graph:



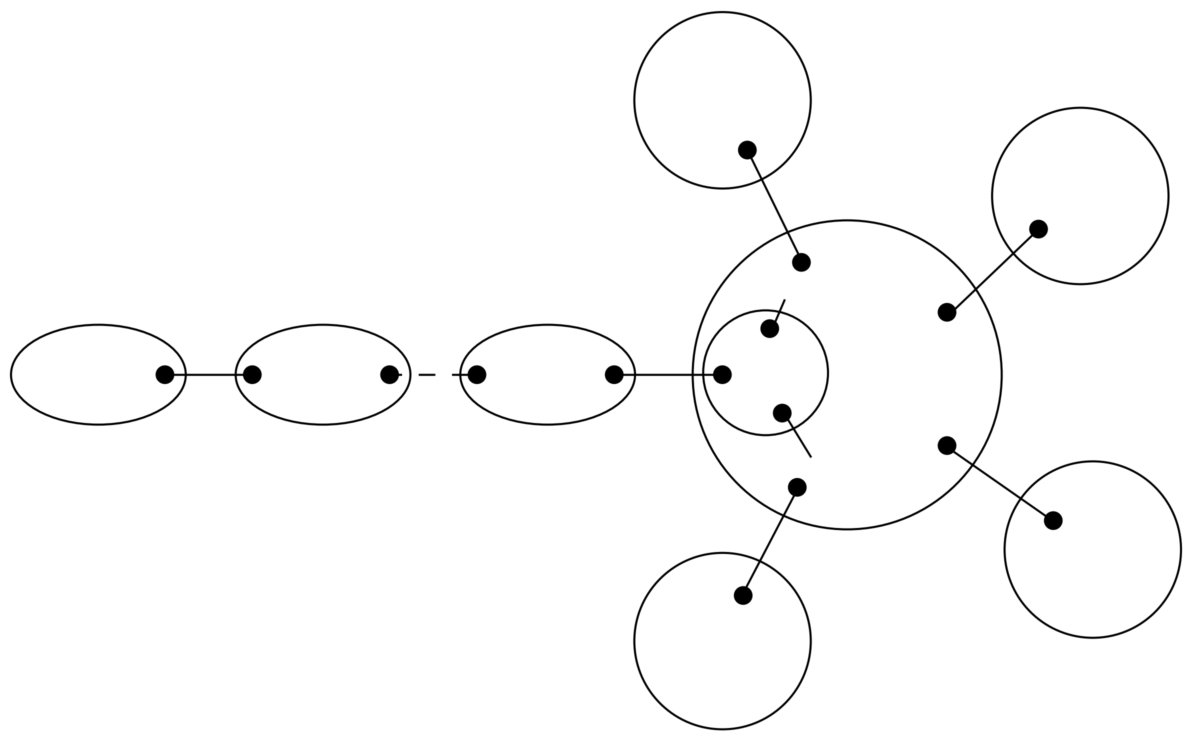
No $(2d+1)$-windmill graph has a d-dimensional MIRS.

Constructing a d-dimensional MIRS

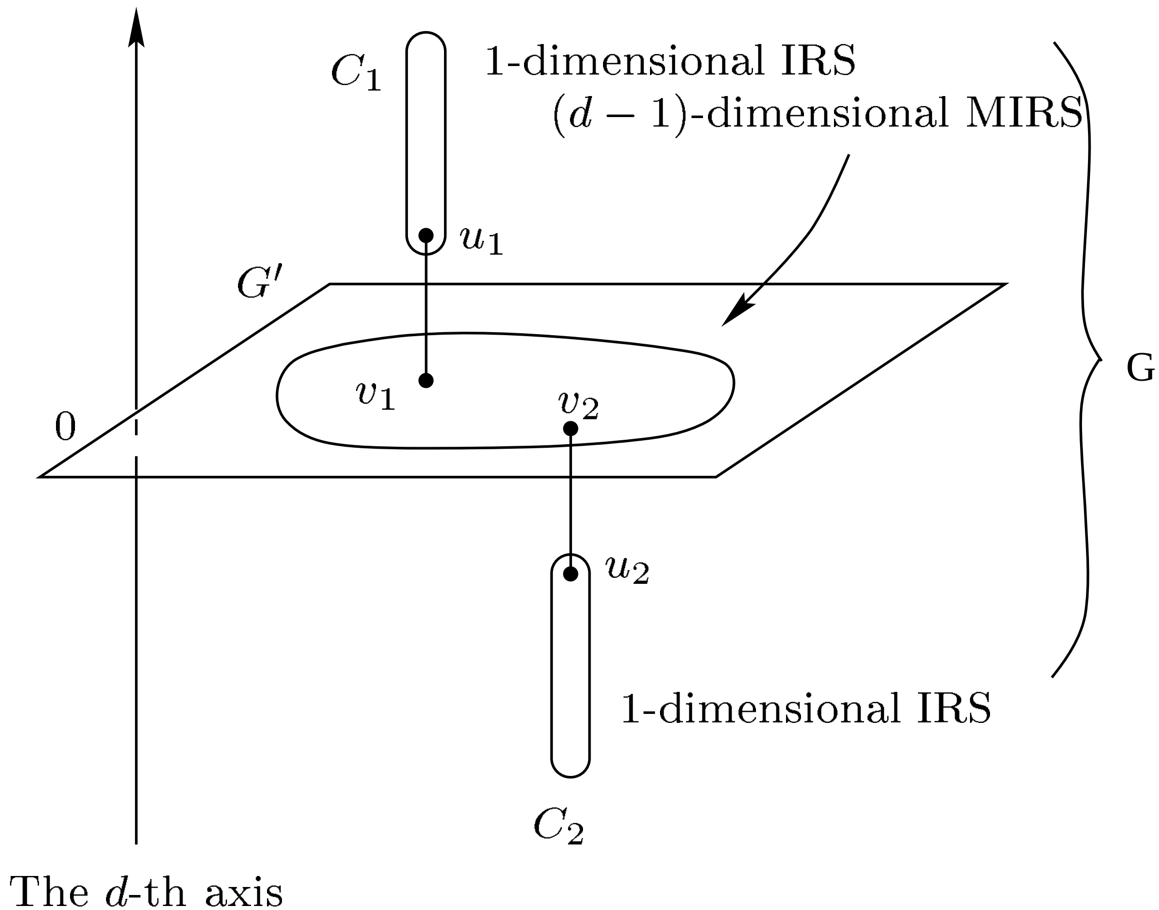
- We use mathematical induction.
- Basis: If a graph is not a 3-windmill graph, then it has a 1-dimensional IRS.
- Hypothesis: If a graph is not a $(2d-1)$-windmill graph, it has a $(d-1)$-dimensional MIRS.
- Want to prove: If a graph is not a $(2d+1)$-windmill graph, it has a d-dimensional MIRS.

Removing a Perfect Chain

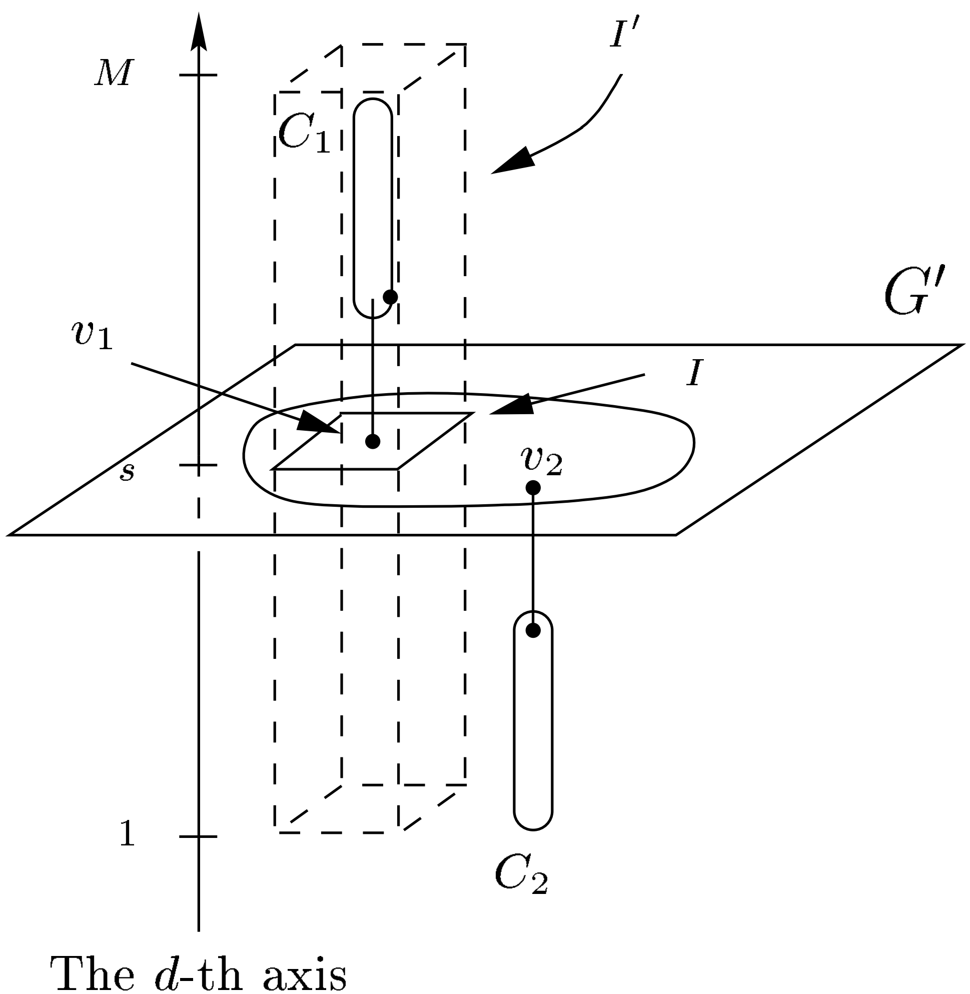
If we remove a perfect chain from a graph which is not a k-windmill graph, the result is not a $(k-1)$-windmill graph.



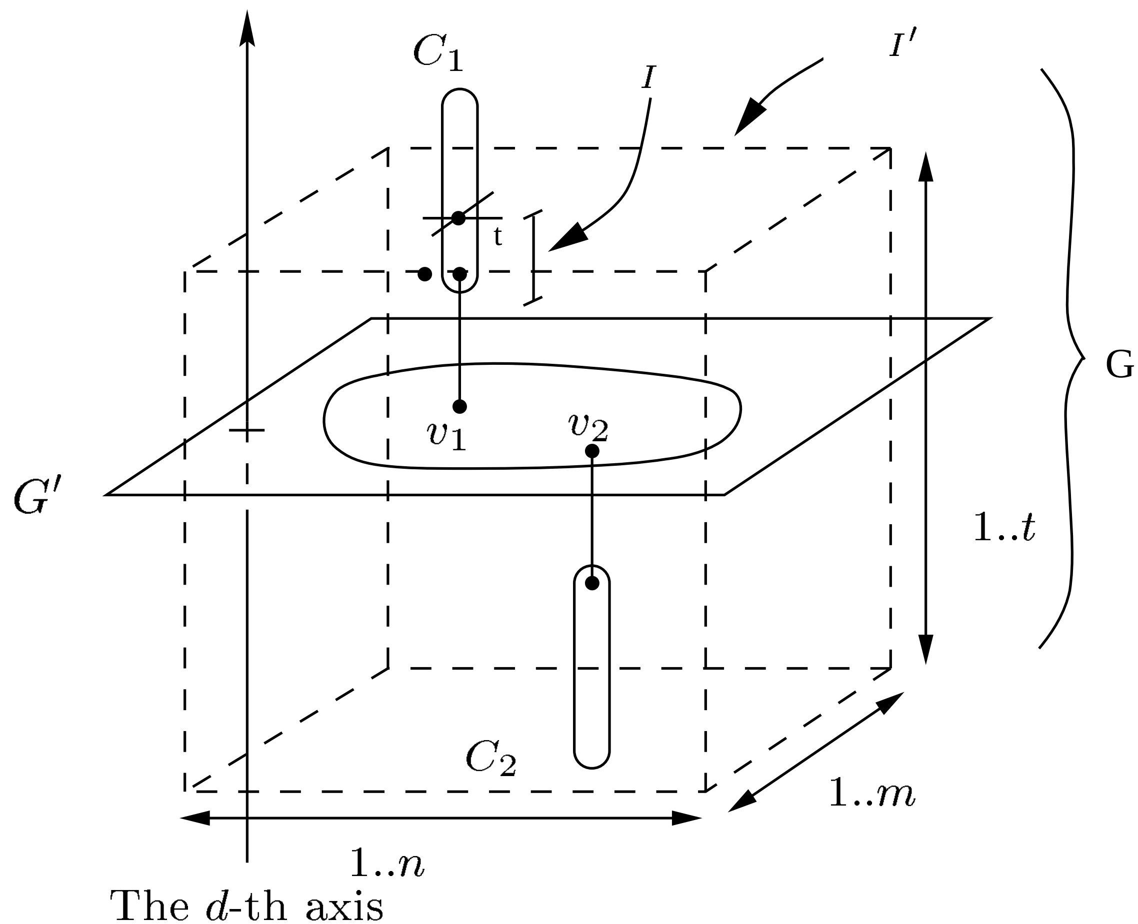
Labeling the Vertices



Updating Intervals



Updating Intervals: Cont'd



Conclusion

1. A graph has a d-dimensional MIRS if and only if it is not a $(2d+1)$-windmill graph. $\star\star\star$
2. A graph has a d-dimensional *strict* MIRS if and only if it is not a *weak* $(2d+1)$-*windmill graph*.