Multi-dimensional Interval Routing Scheme

Master's Thesis Seminar
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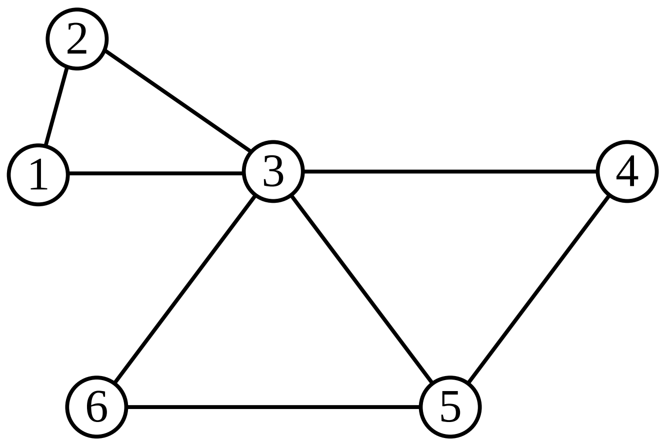
Overview

1. The routing problem and routing schemes
2. Interval routing scheme
3. Evaluation of routing schemes
   - Characterization problem
   - Quality of routing problem
4. Multi-dimensional interval routing scheme (MIRS)
5. Properties of MIRS
6. Conclusion and open problems

The Routing Problem

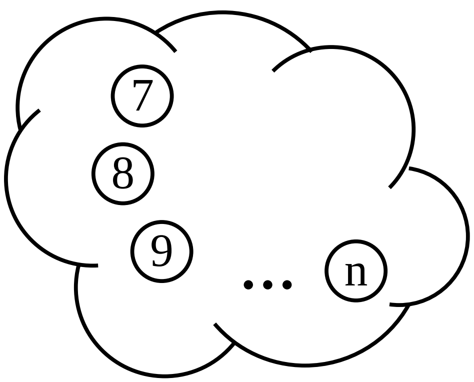


The Classical Solution: Routing Tables

Destination	Forward to
1	3
2	3
3	3
4	--
5	5
6	5

Interval Routing Scheme

Destination	Forward to
[1..3]	3
[5..n]	5



[Santoro and Khatib, 85]

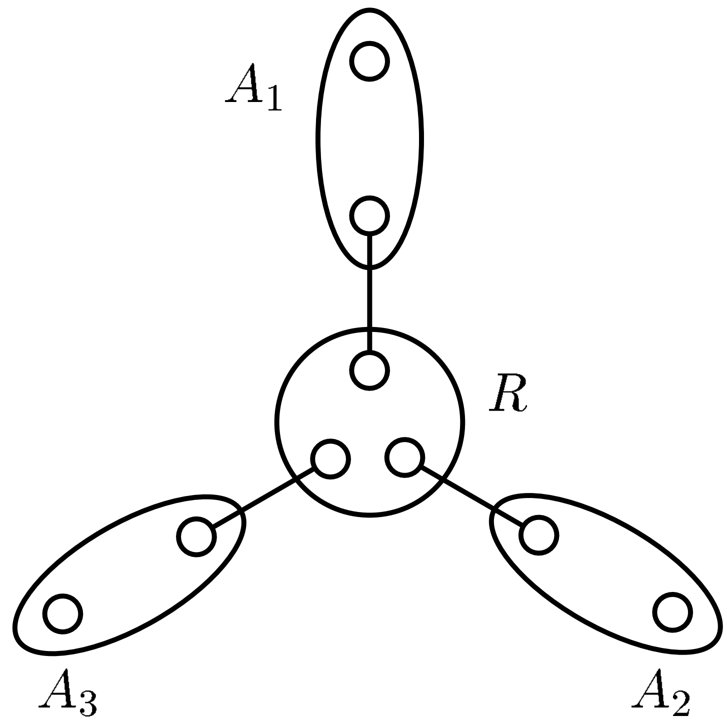
Evaluation of Routing Schemes

1. **Characterization Problem:** Which networks support an IRS? $\star\star\star$
2. **Quality of Routing:** Assuming that a specific network supports IRS, how *good* are the paths traversed by messages?
   - The processing time at each intermediate router.
   - The length of the routing paths. $\star\star\star$

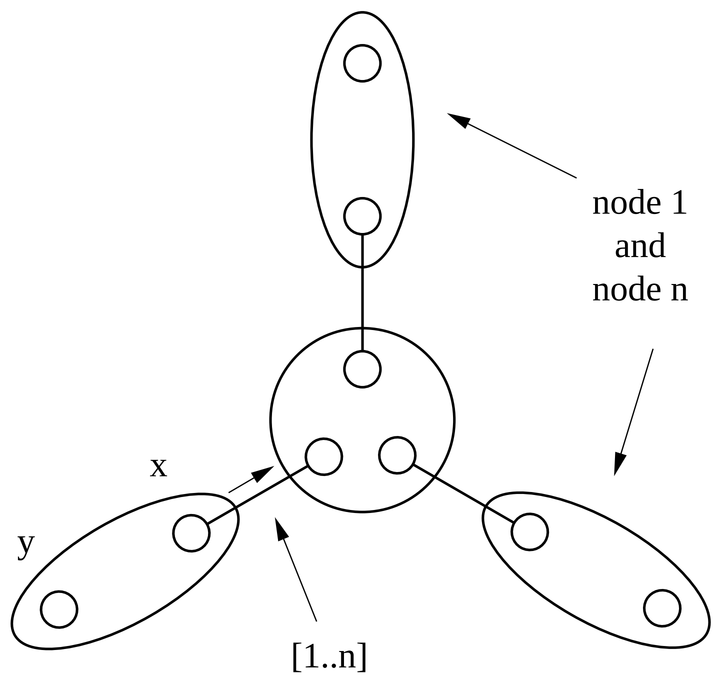
Characterization Problem

A 3-windmill graph:

- has 4 parts (3 *arms* and a *center* R);
- each arm has at least 2 nodes;
- there is no link connecting arms;
- each arm is connected to R with a unique link.



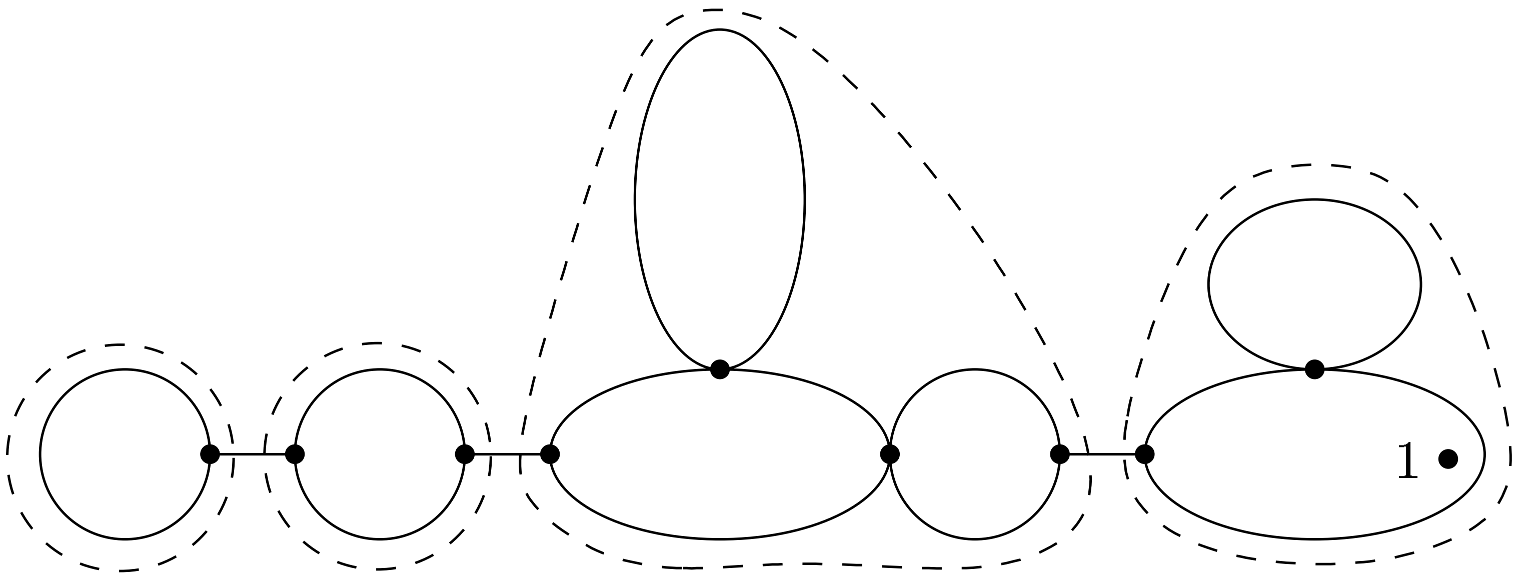
Characterization Problem: Cont'd



A 3-windmill network does not have an IRS.

Characterization Problem: Cont'd

Any graph which is not a 3-windmill graph (chain) has an IRS [Fraigniaud and Gavoille, 94].



G_1 (head) G_2 G_3 G_4 (tail)

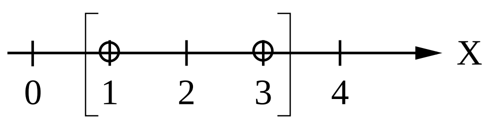
Characterization Problem: Cont'd

How to expand the class of networks which support an IRS?

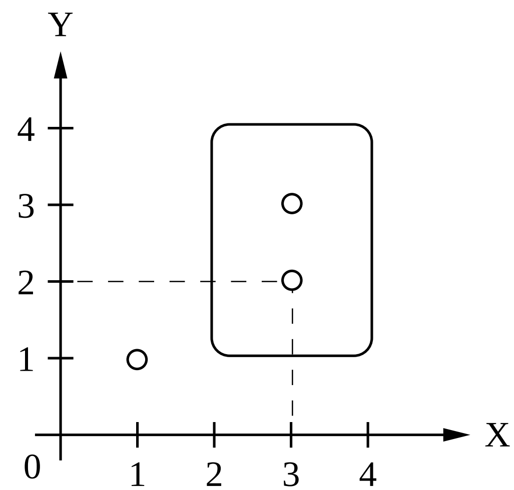
- We can assign *two or more intervals* to each link [van Leeuwen and Tan, 87].
- We can use *multi-dimensional labels* [Flammini *et al.*, 98]. ⋆⋆⋆

Multi-dimensional Interval Routing Scheme

- Node labels: A list of d integers, *e.g.* (3,2)
- Link labels: A list of d intervals, *e.g.* [2..4, 1..2]



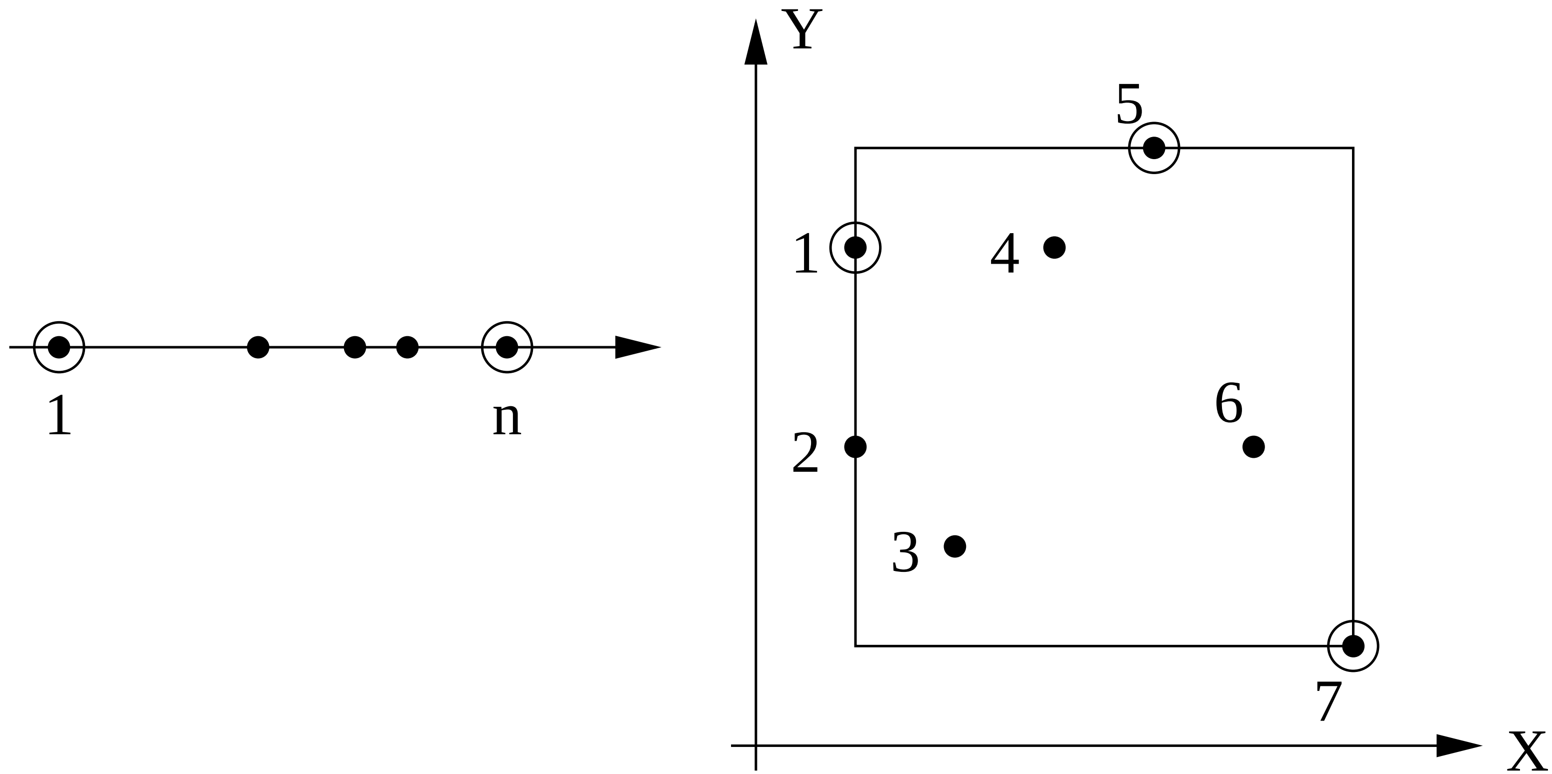
1-dimensional IRS



2-dimensional IRS

A d-dimensional MIRS requires $O(d\Delta n \log n)$ space.

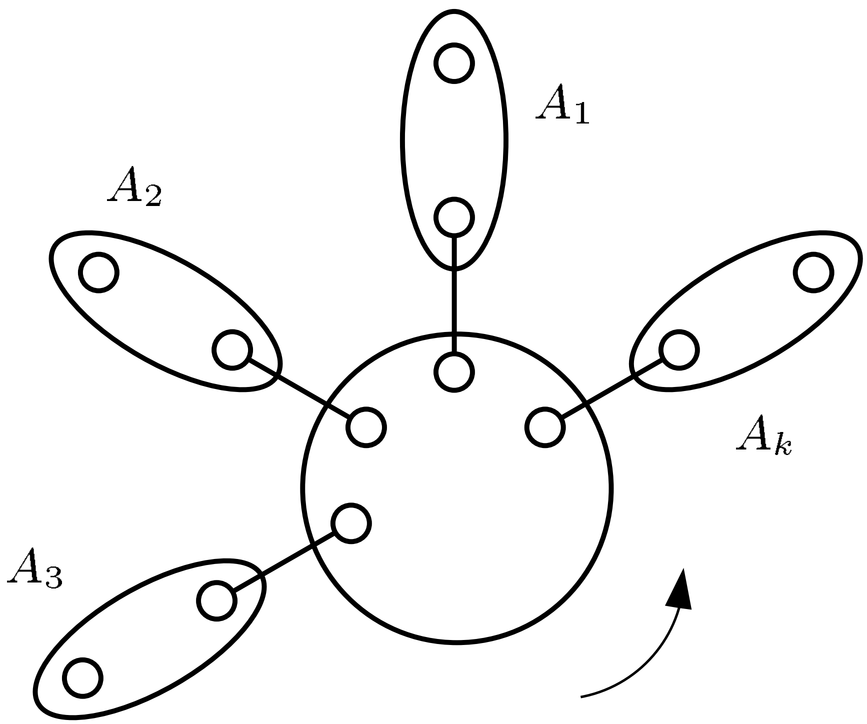
The Boundary Set



Any interval containing the boundary set contains all other points.

Characterization Problem: Cont'd

A k-windmill graph:



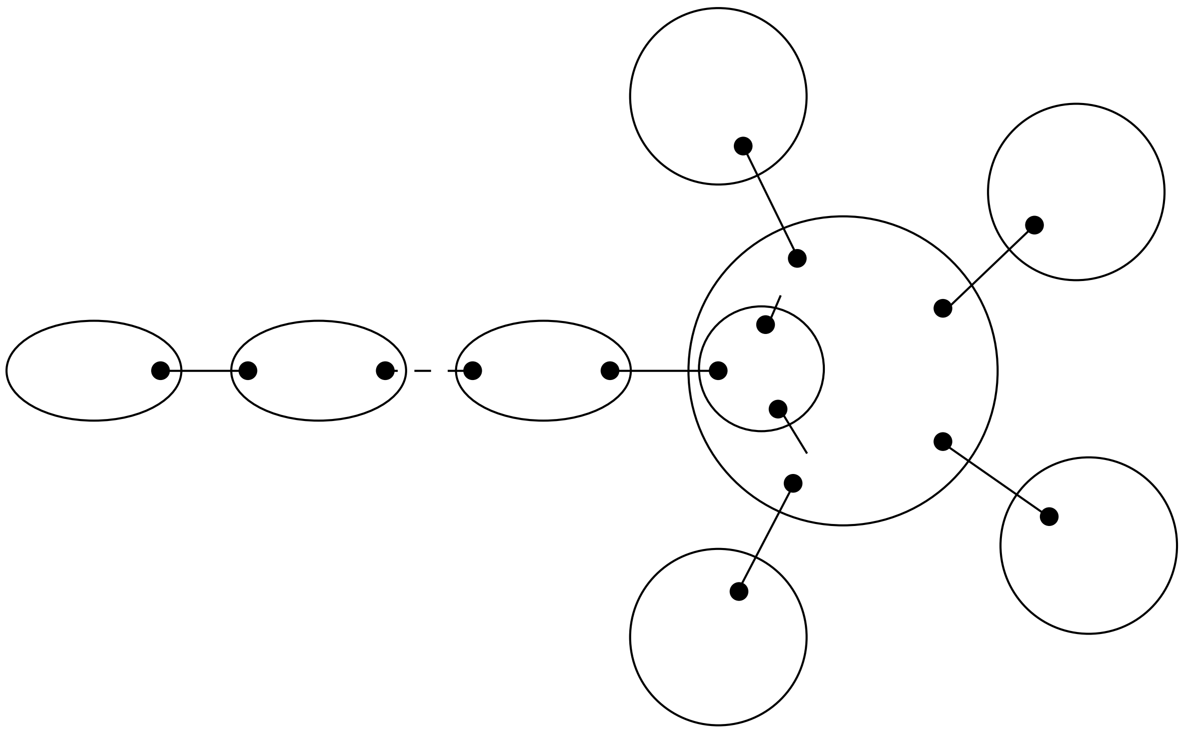
Any $(2d+1)$-windmill graph does not have a d-dimensional MIRS.

Constructing a d-dimensional MIRS

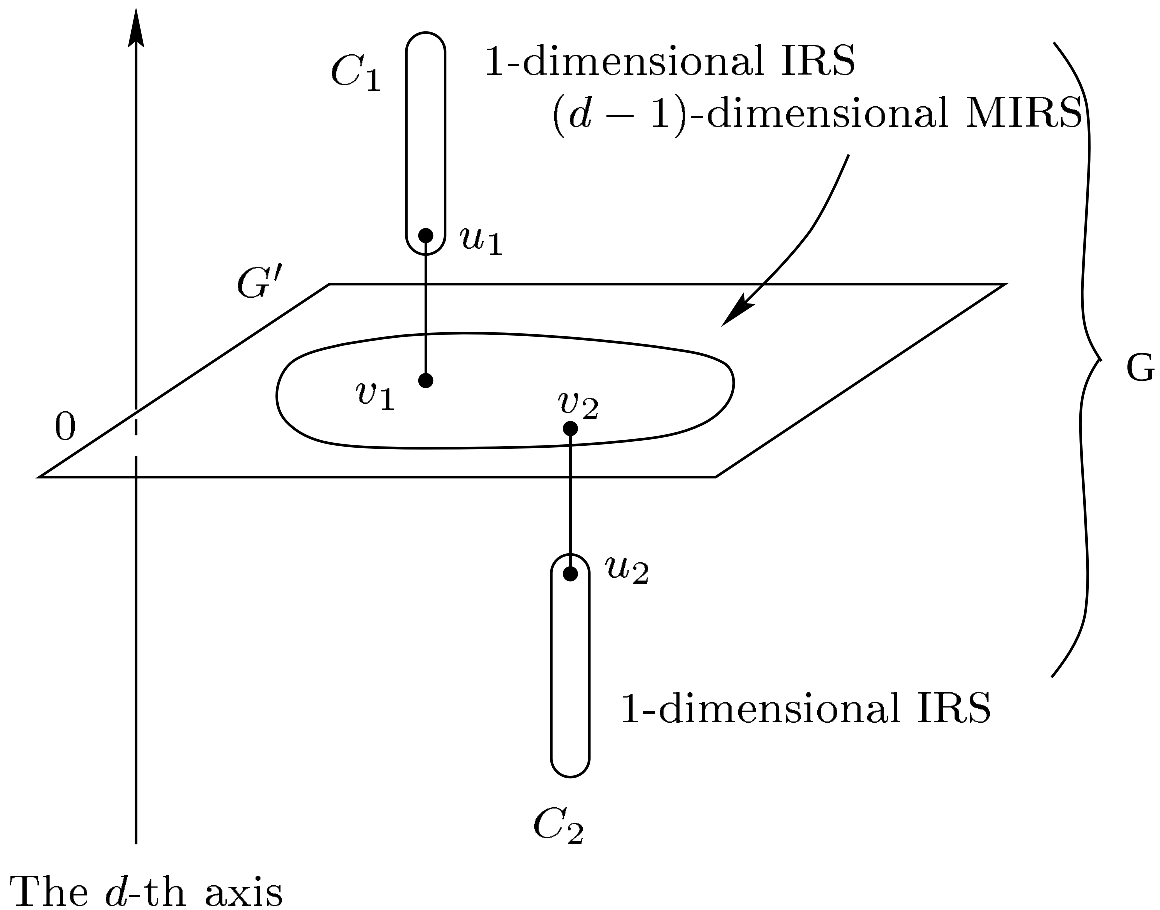
- We use mathematical induction.
- Basis: If a graph is not a 3-windmill graph then it has a 1-dimensional IRS.
- Hypothesis: If a graph is not a $(2d-1)$-windmill graph it has a $(d-1)$-dimensional MIRS.
- Want to prove: If a graph is not a $(2d+1)$-windmill graph it has a d-dimensional MIRS.

Removing a Perfect Chain

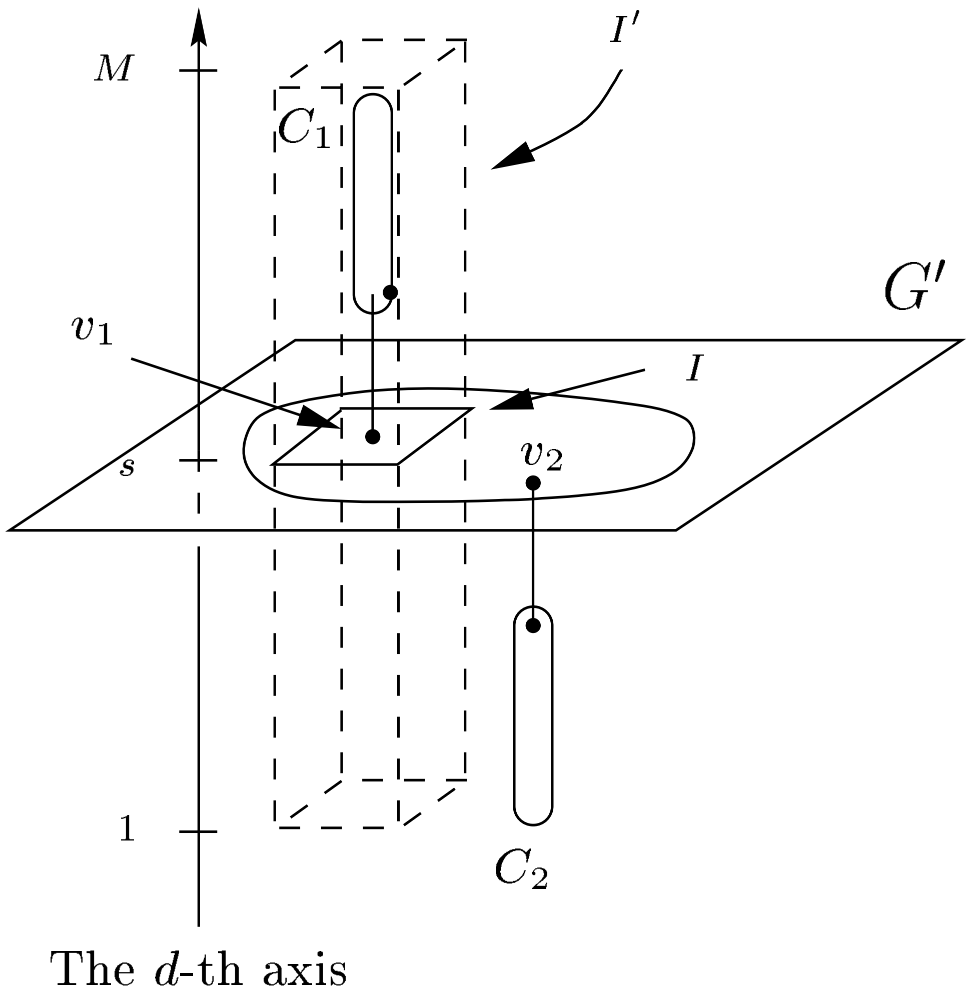
If we remove a perfect chain from a graph which is not a k-windmill graph, the result is not a $(k-1)$-windmill graph.



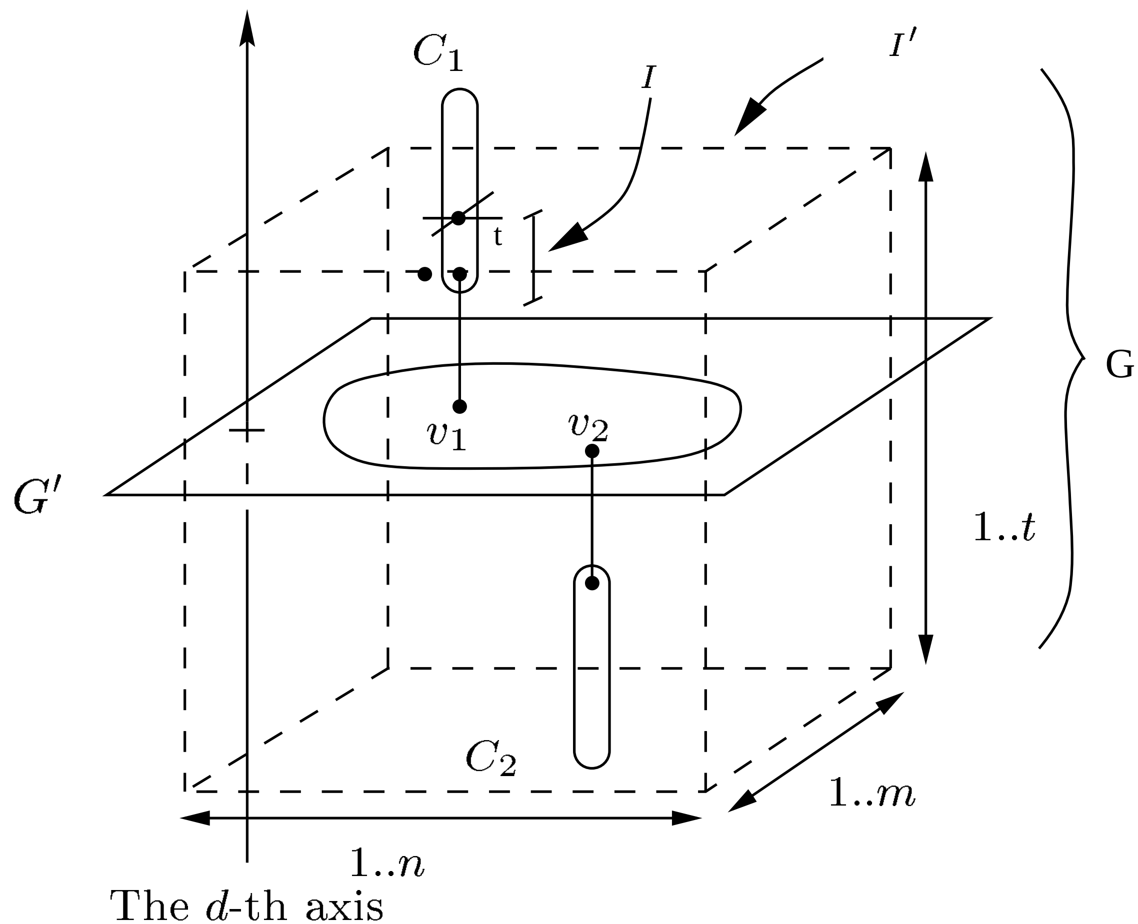
Labeling the Vertices



Updating Intervals

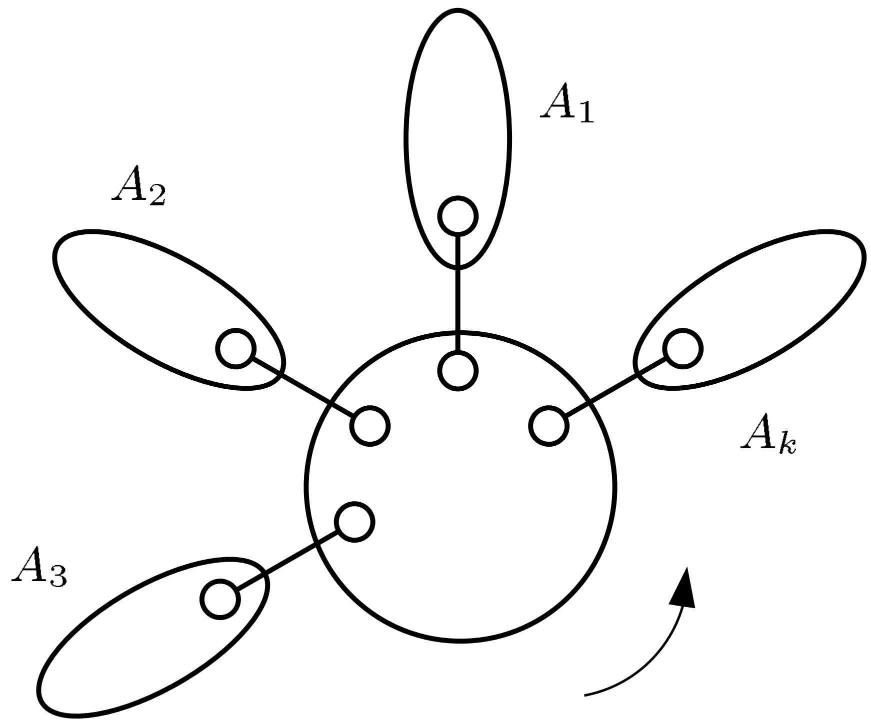


Updating Intervals: Cont'd



Characterization Results: Overview

1. A graph has a d-dimensional MIRS if and only if it is not a $(2d+1)$-windmill graph. $\star\star\star$
2. A graph has a d-dimensional *strict* MIRS if and only if it is not a *weak* $(2d+1)$-*windmill graph*.

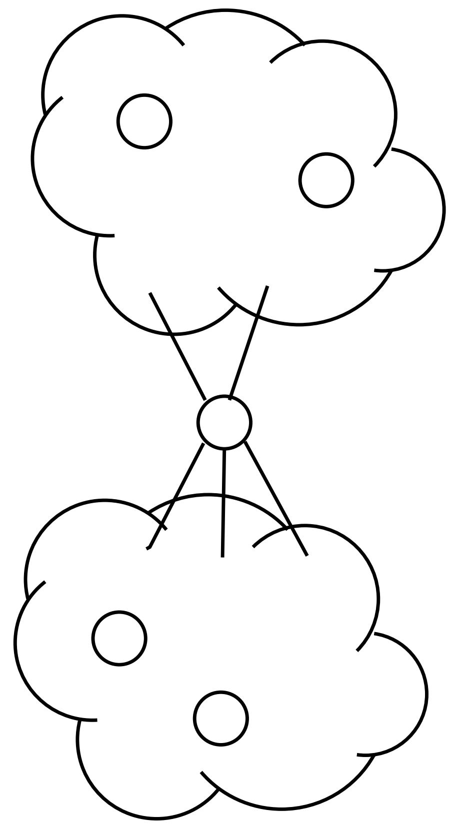


Optimum MIRS / Dynamic Cost Links

- A network is said to have *dynamic cost links* if the costs of the links vary over time.
- An *optimum* MIRS is an MIRS which routes messages on shortest paths.
- An *optimum MIRS with dynamic cost links*
  - The underlying network has dynamic cost links.
  - The labels of nodes are fixed.
  - After each change in the cost of links, we can relabel links.
  - Messages are routed on shortest paths.

Characterization Problem: Cont'd

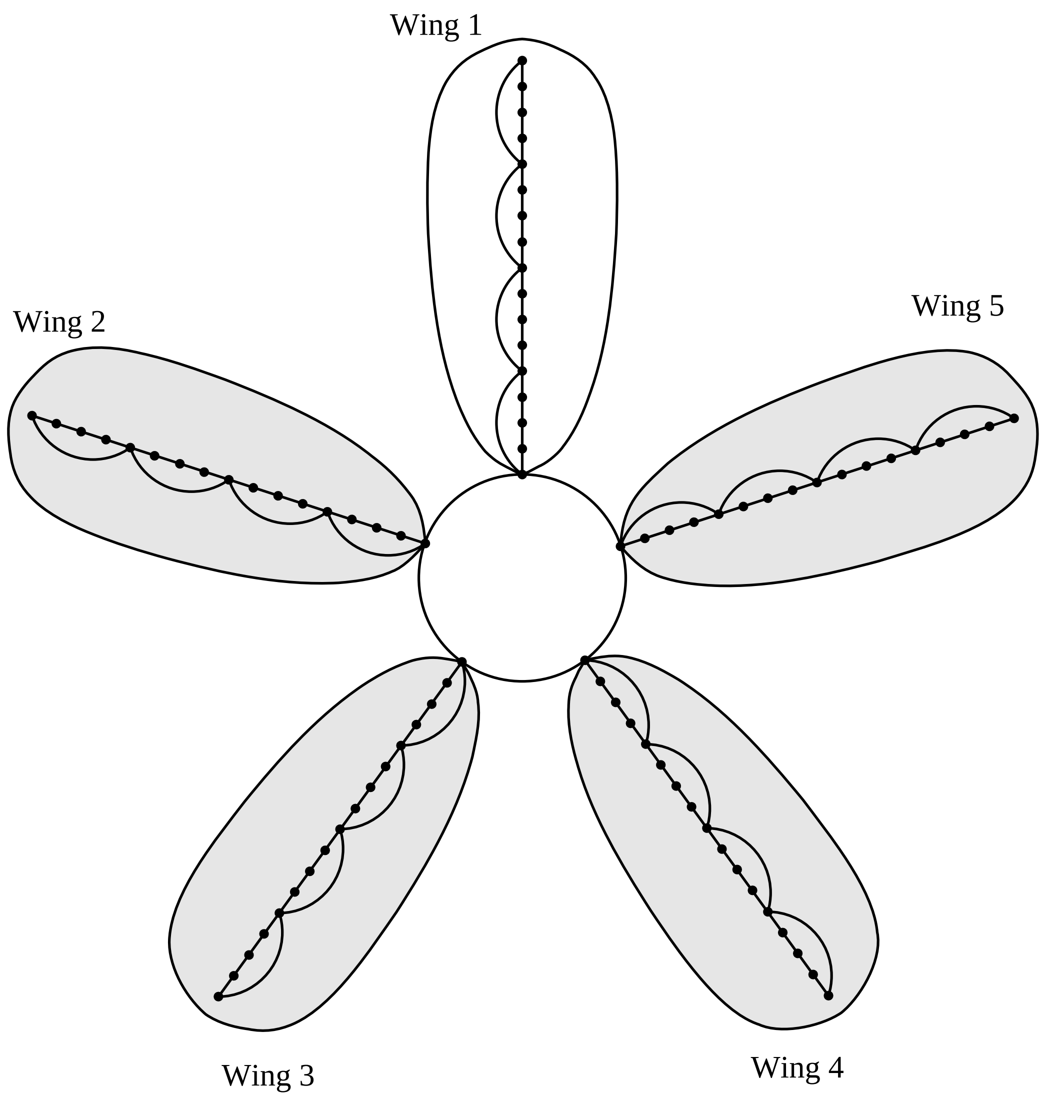
- **Articulation point:** A node whose removal disconnects the network.
- **Non-articulation point:** A node which is not an articulation point.
- **A graph has a d-dimensional optimum MIRS in a network with dynamic cost links if and only if it has at most $2d$ non-articulation points.**



Quality of Routing Problem

- For any $d \geq 1$ there is a graph G such that in any d-dimensional MIRS the length of the longest routing path is in $\Omega(D^2/d)$. $\star\star\star$
- There is a graph G such that for any $\langle k, d \rangle$-MIRS the length of the longest routing paths is at least $\frac{3}{2}D$.
- For any interval graph G there is an IRS such that the length of any routing path is at most $2D - 2$.

Quality of Routing Problem: Cont'd



Conclusion and Open Problems

- Characterization Problem:
  - 1-dimensional *linear* IRS.
  - Specific graphs like hypercubes and grids and rings.
  - Networks supporting d-dimensional linear MIRS.
  - Networks supporting d-dimensional strict and linear MIRS.
  - Networks supporting optimum d-dimensional linear MIRS with dynamic cost links.
- Quality of Routing:
  - 1-dimensional liner IRS.
  - A $\frac{3}{2}D$ lower bound for linear $\langle k, d \rangle$-MIRS.
  - A $\Omega(D^2/d)$ for linear $\langle 1, d \rangle$-MIRS.